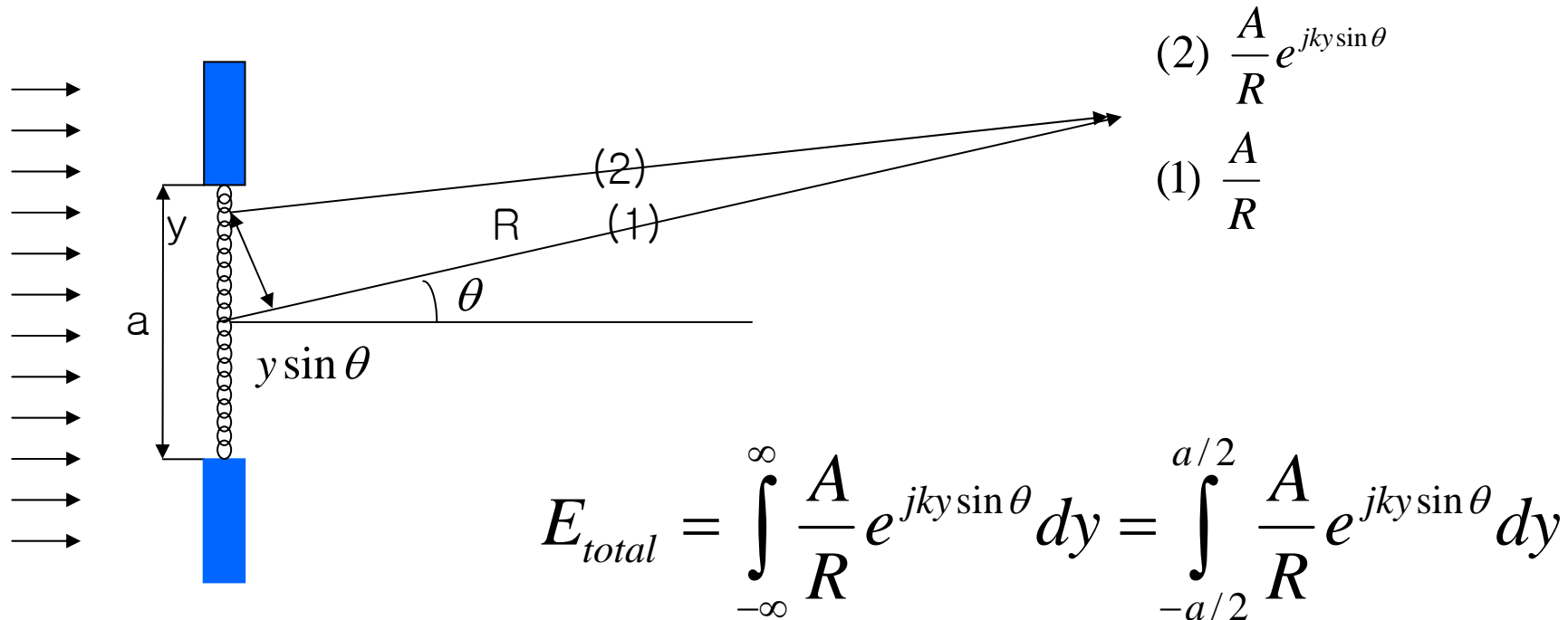


# Lect. 8: Diffraction



# Lect. 8: Diffraction

$$E_{total} = \int_{-a/2}^{a/2} \frac{A}{R} e^{jky \sin \theta} dy$$

$$\text{Let } y' = jky \sin \theta \Rightarrow dy' = jk \sin \theta dy$$

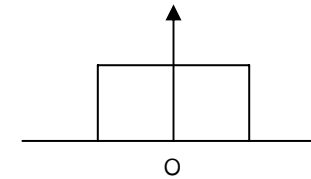
$$E_{total} = \int_{-a/2}^{a/2} \frac{A}{R} e^{y'} \frac{dy'}{jk \sin \theta} = \frac{A}{R} \frac{1}{jk \sin \theta} \left( e^{jk \frac{a}{2} \sin \theta} - e^{-jk \frac{a}{2} \sin \theta} \right)$$

$$= \frac{A}{R} \frac{2j}{jk \sin \theta} \sin\left(k \frac{a}{2} \sin \theta\right)$$

$$= \frac{2A}{R} \frac{\sin\left(k \frac{a}{2} \sin \theta\right)}{k \sin \theta}$$

$$E_{total}(0) = \frac{2A}{R} \frac{\cos\left(k \frac{a}{2} \sin \theta\right) k \frac{a}{2} \cos \theta}{k \cos \theta} = \frac{2A}{R} \frac{a}{2}$$

$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\sin\left(k \frac{a}{2} \sin \theta\right)}{k \frac{a}{2} \sin \theta} = \frac{\sin \psi}{\psi} \quad \left( \psi = k \frac{a}{2} \sin \theta \right)$$



sinc function

Fraunhofer Diffraction:  $R \gg \lambda$

FT relationship

# Lect. 8: Diffraction

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Remember

$$f(t) = \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df \quad f : y$$

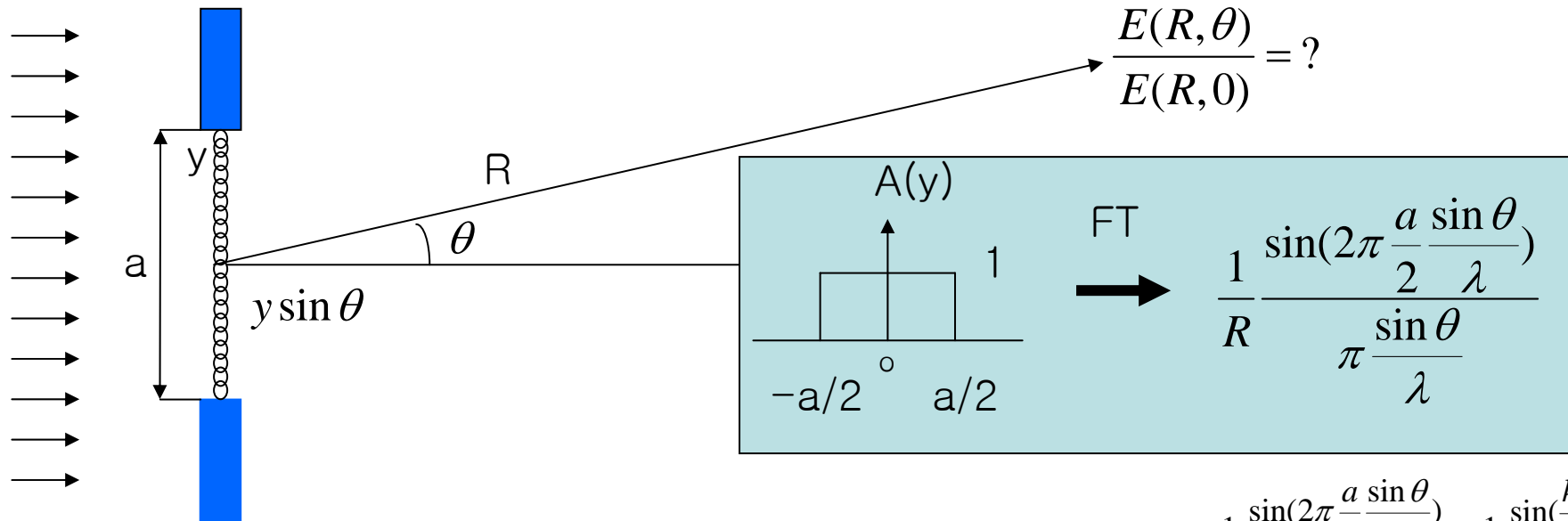
$$E_{total} = \int_{-\infty}^{\infty} \frac{A(y)}{R} e^{jk_y \sin \theta} dy \quad t : \frac{\sin \theta}{\lambda}$$

$$E_{total} \left( \frac{\sin \theta}{\lambda} \right) = \int_{-\infty}^{\infty} \frac{A(y)}{R} e^{j2\pi y \frac{\sin \theta}{\lambda}} dy$$

$$f(t) \quad \Leftrightarrow \quad F(f)$$

$$E\left(\frac{\sin \theta}{\lambda}\right) \Leftrightarrow A(y)$$

# Lect. 8: Diffraction

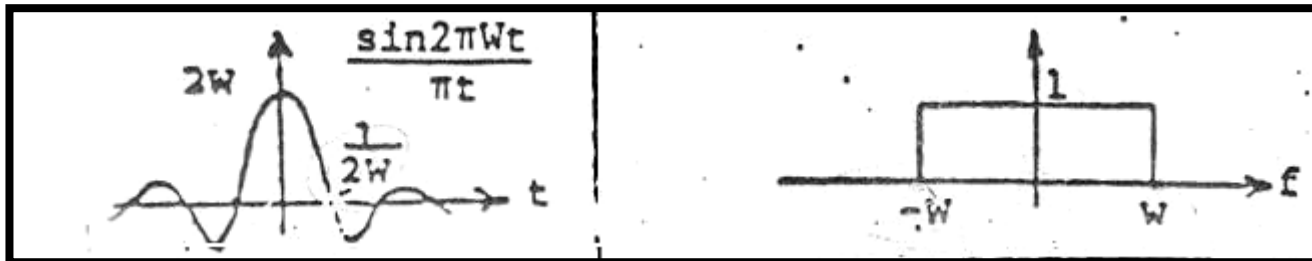


$$\frac{1}{R} \frac{\sin(2\pi \frac{a \sin \theta}{2 \lambda})}{\pi \frac{\sin \theta}{\lambda}} = \frac{1}{R} \frac{\sin(\frac{ka}{2} \sin \theta)}{\frac{k}{2} \sin \theta}$$

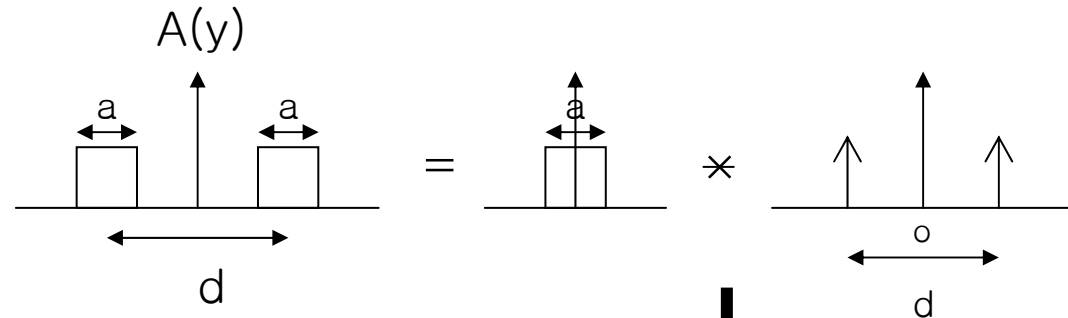
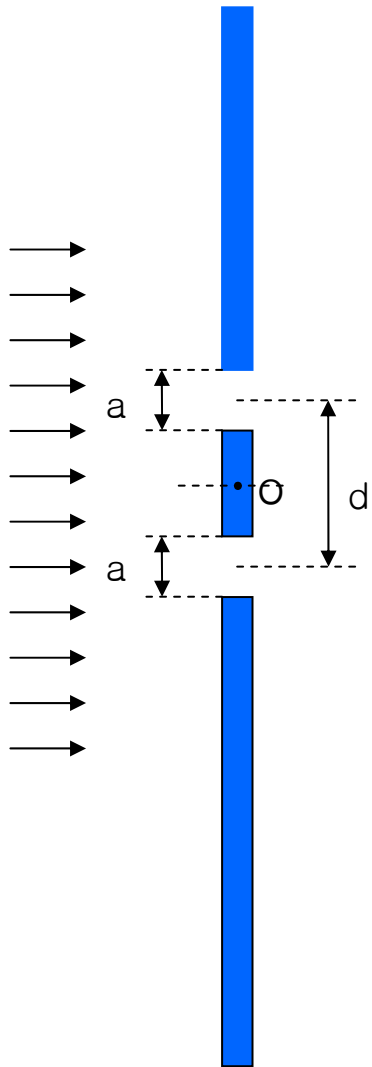
Since  $E(\theta = 0) = \frac{a}{R}$ ,

$$\frac{E(\theta)}{E(0)} = \frac{\sin(\frac{ka}{2} \sin \theta)}{\frac{ka}{2} \sin \theta}$$

$f(t)$  (From Signals textbook)  $F(f)$



# Lect. 8: Diffraction

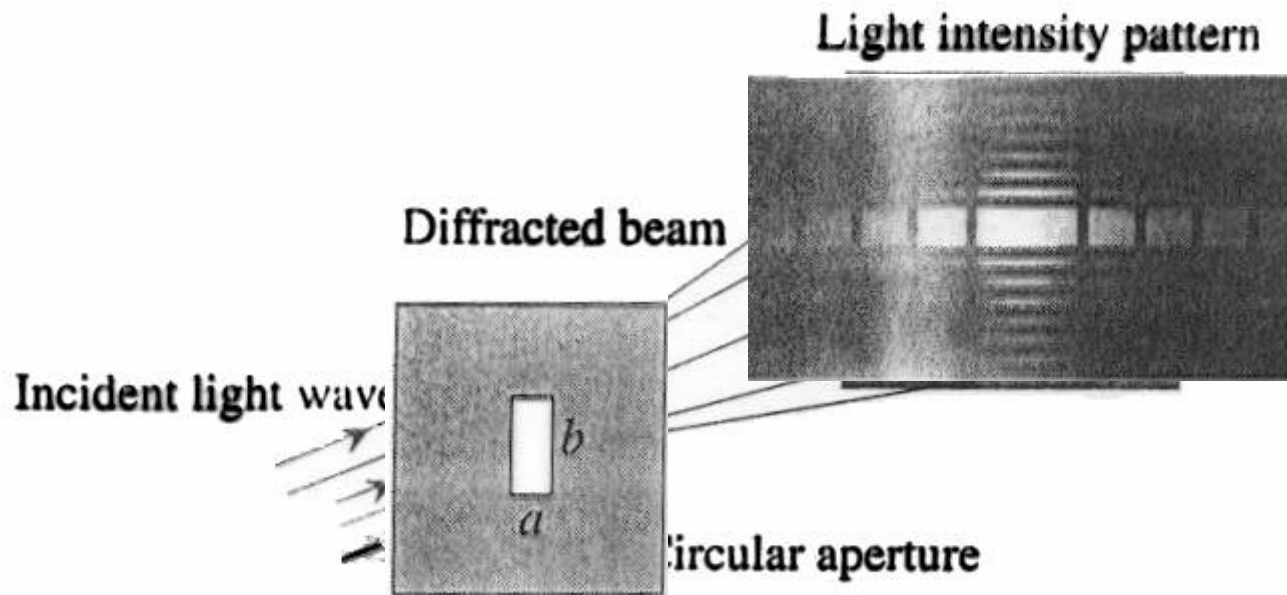


$$\frac{1}{R} \frac{\sin\left(2\pi \frac{a \sin \theta}{2 \lambda}\right)}{\pi \frac{\sin \theta}{\lambda}} \times \left( e^{j2\pi \frac{d \sin \theta}{2 \lambda}} + e^{-j2\pi \frac{d \sin \theta}{2 \lambda}} \right) = 2 \cos\left(2\pi \frac{d \sin \theta}{2 \lambda}\right)$$

$$\frac{E(\theta)}{E(0)} = \cos\left(k \frac{d \sin \theta}{2}\right) \frac{\sin\left(\frac{ka}{2} \sin \theta\right)}{\frac{ka}{2} \sin \theta}$$

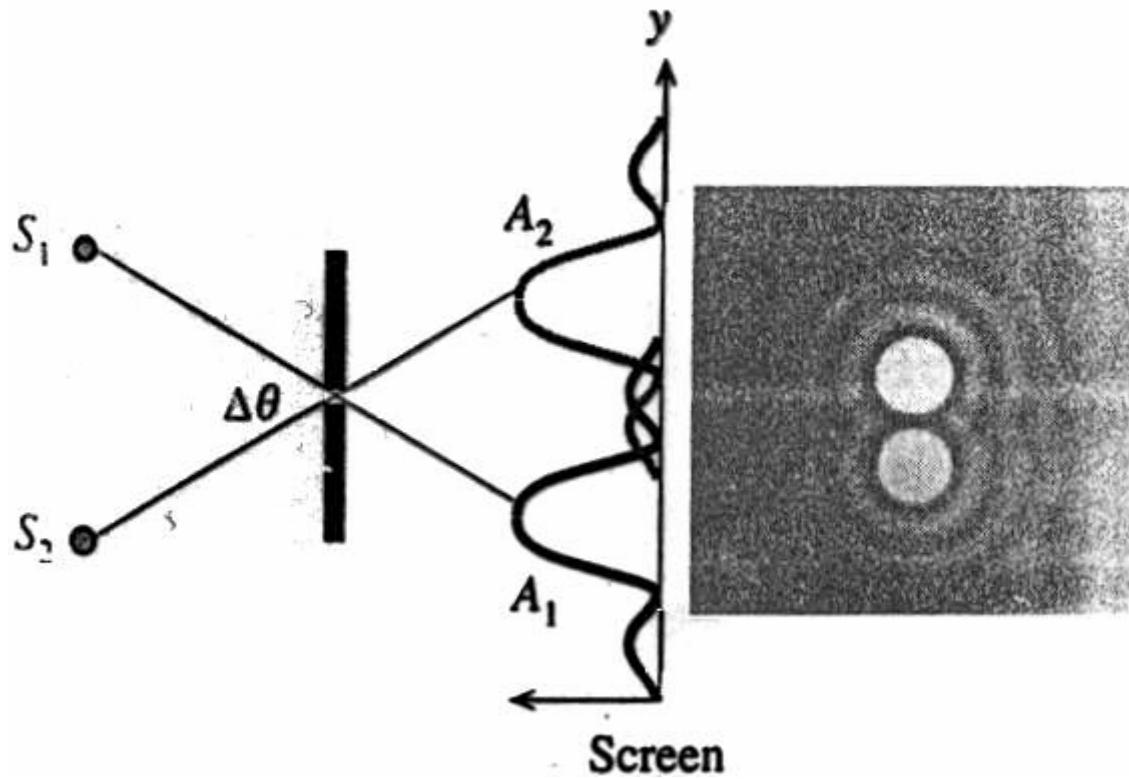
# Lect. 8: Diffraction

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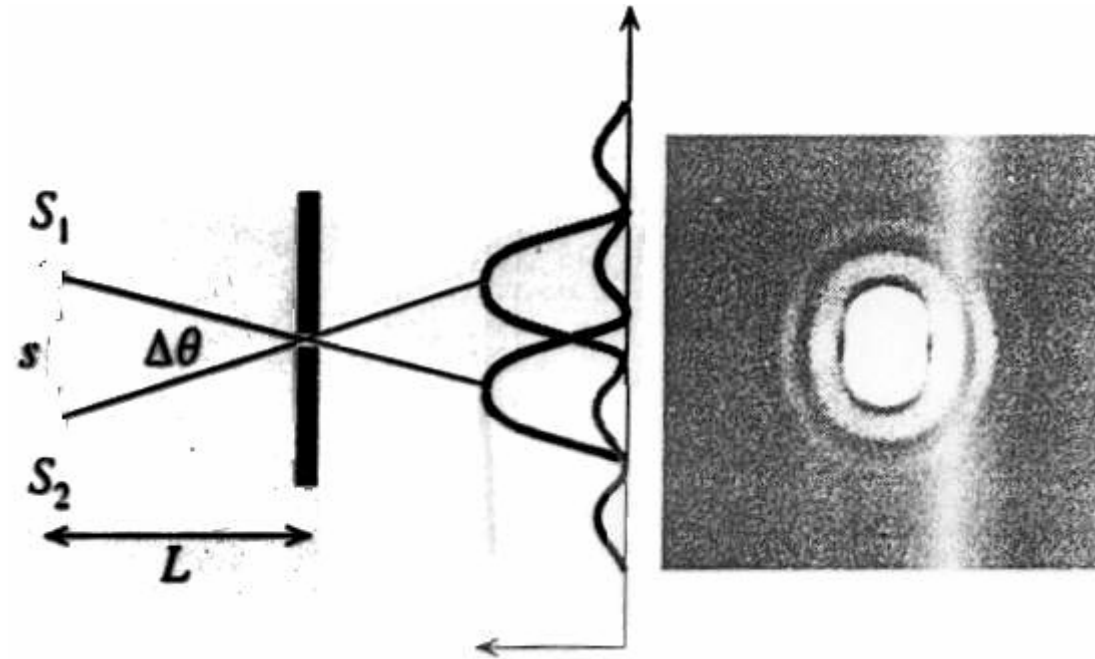
# Lect. 8: Diffraction

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# Lect. 8: Diffraction

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$$\Delta\theta \sim \frac{1}{a}$$